Salvo Model
for
Anti-Surface Warfare Study

Ed Hlywa
Weapons Analysis LLC

In the late 1980's Hughes brought combat modeling into the missile age by developing an attrition model inspired by the exchange of striking power against defensive power during the World War II Battle of Midway. His salvo exchange model described combat as a pulse of offensive combat power, measured in number of Anti-Ship Cruise Missiles (ASCMs), attempting to penetrated active defenses to cause damage to the opposing ships.

While lacking the complexity of modern day models, Hughes' Salvo Model is a useful analytic tool for assessing the crucial capabilities of surface combatants. The model provides a method to analyze trade-offs between defensive power, offensive power, ship vulnerability, and force size. The model can be used to compute the fractional exchange ratio of one force to another based upon the number of missiles that continue towards their targets after having been engaged by defensive systems of the opposing force.

Although the model has impressive descriptive power, two major drawbacks exist: the model's predictive powers are limited to homogeneous forces, and the model is strictly deterministic. In modern day warfare, homogeneous forces would be rare, and deterministic and instantaneous attrition obscures the importance of sensing the presence of opposing forces and sequencing through the kill chain.

These shortfalls have been addressed by introducing model variants for heterogeneous forces and stochastic events. However, the utility of these models is questionable. The heterogeneous variant (Johns, 2000) requires a high-dimensional "matching matrix" to define every interaction between elements of offensive power, defensive power, and ship vulnerability. A stochastic version (McGunnigle, 1999) that accounts for the variability of of offensive and defensive powers, only works for homogeneous forces. A model that combines both the stochastic and heterogeneous attributes would be largely academic and impractical. In short, a full description of the matching matrix would be tantamount to an a-priori description of all salvo exchanges and would obviate the need for the model to begin with (Cares, 2004).

Other variations of Hughes' Salvo Model that maintain the simplicity of the basic equations while providing higher fidelity are found in numerous Navy studies and graduate theses. Variations include features such as: providing different levels of offensive and defensive capability (Hughes, 1995), accounting for the effects of ASCM leakers (Tiah, 2007), modeling the effects of soft-kill measures, and providing the capability to re-engage with anti-ASCM missiles (Wissel, 2008). While incorporating greater detail is an worthwhile exercise, it is important to note that higher fidelity representations of the basic terms of the salvo model do not change the outcome of the model. Higher fidelity is just a convenient way to discriminate between the capabilities of the opposing forces.

For the Anti-Surface Warfare Study, the salvo model is supplemental to the primary analytical tool, Naval Simulation Systems (NSS). Therefore, judicial choices of the salvo model's capability and fidelity must be weighed against its intended use. The model's primary use is to limit the analysis space that will be probed by NSS. While a-priori requirements for such a salvo model cannot be established, it is reasonable to assume that the basic form of the salvo model, along with a reasonable level of fidelity of its terms, will be sufficient to guide a detailed exploration with NSS.

The Anti-Surface Warfare Study also starts with the premise that Blue is "out sticked” and “out-ranged.” It is obvious that ASCM defense will be a critical factor in Blue's ability to survive the first strike. Consequently, it is reasonable to assume that higher fidelity in the defensive-power terms of the salvo model will be a good thing to include.

This paper describes the features of the salvo model developed for the Anti-Surface Warfare Study. It begins with a description of the basic salvo model by Hughes, and builds upon that foundation with motivation for features that support the study objectives.

Hughes' Salvo Model - Basic Form

Hughes' basic Salvo Model (Hughes,1992) calculates the fraction of opposing forces taken out of action as a result of ASCM salvos being fired by one force and defended against by the opposing force. The basic equations for two forces, A and B, are given by:

$$\Delta B = \frac{\alpha A - b_3 B}{b_1} \qquad \Delta A = \frac{\beta B - a_3 A}{a_1}$$

where:

ΔA = Number of ships in force A taken out of action by B's salvo.
ΔB = Number of ships in force B taken out of action by A's salvo.

and:

A,B = Number of ships in forces A,B.
α,β = Number of ASCMs fired by each ship in forces A, B.
a_1, = Number of hits by B's ASCMs needed to put a ship in force A out of action.
b_1 = Number of hits by A's ASCMs needed to put a ship in force B out of action.
a_3 = Number of B's ASCMs defeated by the defenses of each ship in force A.
b_3 = Number of A's ASCMs defeated by the defenses of each ship in force B.

Hughes' equations are symmetrical. Each equation represents one of the opposing forces. In each equation, the first term in the numerator represent the number of ASCMs launched at the opposing force. This is the Striking Powers of the force. The other term in the numerator represents the number of ASCMs that are defeated by the defensive systems of the opposing force. This is the Defensive Power of the force. The difference between the Striking Power of one force and the Defensive Power of the opposing force is the number of ASCMs that will damage the defending force. This called the Combat Power of the force. Scaling the number of ASCMs that damage a force by the vulnerability of the ships in that force, results in the number of ships that are taken out of action. The vulnerability of the ships, measured in numbers of ASCMs required to take a ship out of action, is called the Staying Power of the force.

So, for forces A and B:

$$B \text{ Out of Action} = \frac{\text{Striking Power of Force } A - \text{Defensive Power of Force } B}{\text{Staying Power of Force } B}$$

and

$$A \text{ Out of Action} = \frac{\text{Striking Power of Force } B - \text{Defensive Power of Force } A}{\text{Staying Power of Force } A}$$

Hughes' Salvo Model makes the following assumptions:

- The salvo exchange and ASCM defense, between the opposing forces, occurs simultaneously
- ASCMs are uniformly distributed over the opponent's force.
- The defensive capability is uniformly distributed over the opponent's force.
- Hits on a force will decrease the effectiveness of a ship linearly until it is out of action.

The basic form of Hughes' model is deceptively simple even though the interactions between the forces are complex. The simplicity of the model contradicts the complexity of the interactions between the principle factors.

Development of the Salvo Model

The Salvo Model for the Anti-Surface Warfare Study builds on the basic form of Hughes' model, and draws on the works of others (Tiah 2007 and Wissel 2008) to encompass modeling attributes that are applicable to the study's objective.

While the Anti-Surface Warfare Study Model retains the basic form of Hughes' Salvo Model, several important enhancements distinguishes it from previous works. The Anti-Surface Warfare Study Salvo Model includes the following features:

- Striking Power and Defensive Power are refined to provide greater discrimination of force capabilities.
- Offensive decoys that dilute the effectiveness of SAM defenses are modeled.
- ASCM leakers that penetrate the SAM defenses are modeled.
- Soft-kill defenses are included as the middle layer of ASCM defense.
- Point defense systems are modeled as the inner layer of the ASCM defense.

The sequence of the model's features is shown in Figure 1. The figure actually shows one half of the model. An identical flow diagram, representing the opposing force, is implied by the Blue/Red color scheme.

The figure shows the offensive power of one force (Blue) challenging the defensive layers of the opposing force (Red). The defensive layers are modeled as a Markov process whereby the success of one layer is based on the outcome of the previous layer. It is convenient to define the number of surviving ASCMs as the Combat Power of the attacking force and to recompute the Combat Power at each layer of the ASCM defense. The final value of the Combat Power (after progressing through the layered defenses) is the number of ASCMs that will damage the ships of the opposing force. This number, scaled by the Staying Power of the force, gives the number of ships that were taken out of action.

The following sections describes the salient points of each of the model's functions.

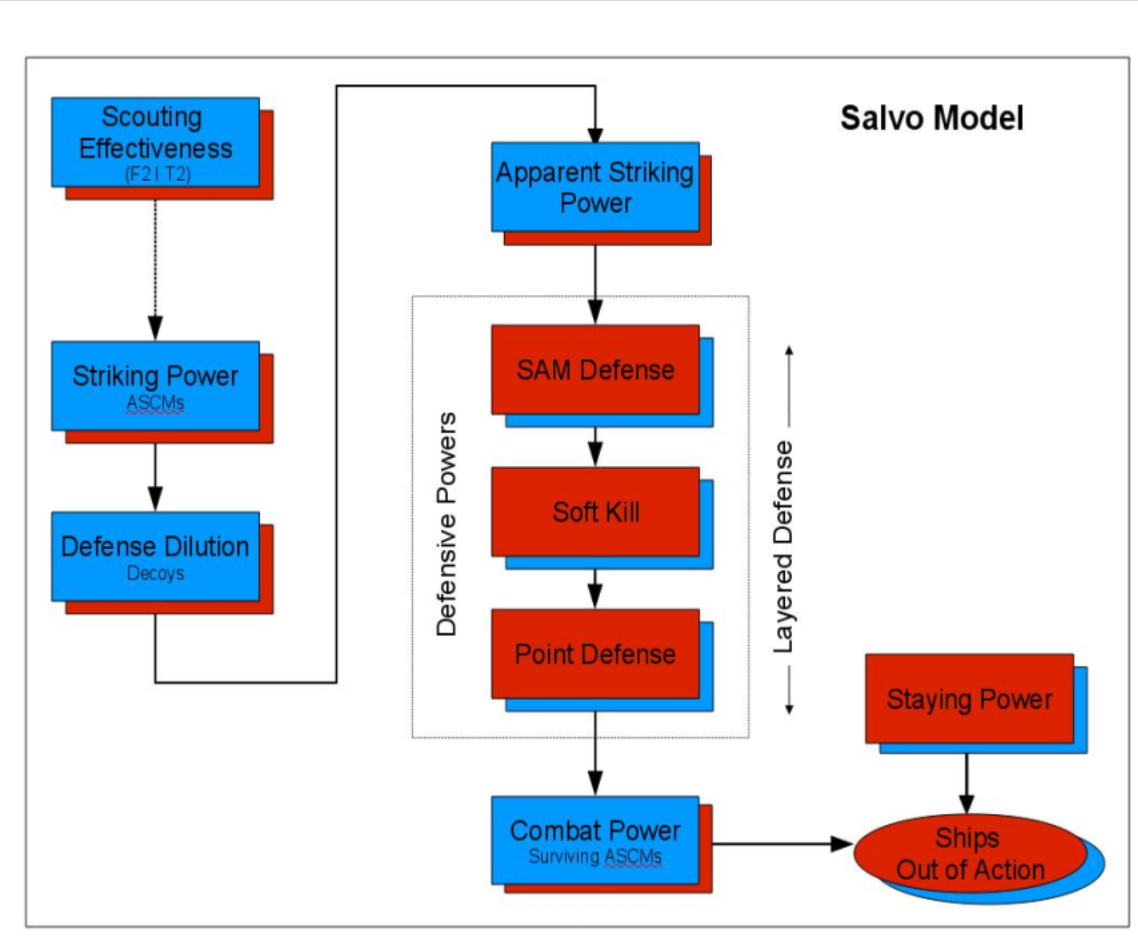


Figure 1 Salvo Model Function Flow Diagram

Striking Power

Striking Power is the measure of the offensive pulse of the salvo exchange. It is the number of ASCMs directed the opposing force. The Anti-Surface Warfare Salvo Model refines Striking Power to be the product of the following data:

- Scouting Effectiveness of the force
- Average ASCM Salvo Size
- Average ASCM Launch Reliability
- Average ASCM Hit Probability
- Number of Ships in the force

Scouting Effectiveness refers to the ability of the force to collect and process all the essential information about the enemy to effectively attack it. It should be considered as an aggregate measure of effectiveness for the Find, Fix, Identify, Track and Target elements of the kill chain[1]. Scouting Effectiveness has a value between zero and one. A value of zero means that there is no information about the enemy and no ability to hit any targets. A value of one

[1] The kill chain is a military concept involving the sequential and unbroken chain of events resulting in the destruction of the target. Typical chain events are: Find (target), Fix, Identify, Track, Target, Engage, and Assess.

means the opposing force is within effective range and being tracked, so that every ship may be targeted.

Since the Hughes Salvo Model assumes a homogeneous force structure, the Average ASCM Salvo Size is the number of ASCMs launched by each ship of the force.

ASCM Launch Reliability accounts for less than perfect ability to launch missiles. This includes the effects of mechanical and electrical malfunctions, operator training, combat readiness, and mechanical problems. Both missile and launching systems effects are included in this metric. Launch Reliability is assigned a value between zero and one.

ASCM Hit Probability refers to the probability that an ASCM will hit and damage a targeted warship in the absence of anti-ASCM defenses.

The Number of Ships scales data to the force level. The model assumes that all ships contribute equally to the Striking Power of the force.

Apparent Striking Power

During the course of the Anti-Surface Warfare Study, it was determined that offensive decoys can be used to dilute the defensive effectiveness of the opposing forces. By launching offensive decoys along with ASCMs, the defenses of the opposing force "sees" more threats than actually exist. Since the defending force cannot distinguish between the offensive decoys and ASCMs, it must assume that all inbound missiles are ASCMs, and as such, will deplete its defensive resources by engaging less than lethal targets. The total number of ASCMs and decoys delivered to the opposing force is called the Apparent Striking Power.

The Anti-Surface Warfare Salvo model assumes that offensive decoys share the same attributes of ASCMs. Offensive decoys have the same launch reliability as ASCMs, and the probability of an offensive decoy flying to the proper location to entice the SAM defenses is the same as the probability of an ASCM hitting its intended target. Both ASCM and offensive decoy launches are based on the same scouting effectiveness.

SAM Defense

The SAM defense strategy attempts to assign a fire control channel to each ASCM and decoy in the salvo. The Anti-Surface Warfare Salvo Model assumes each fire control channel will successfully defeat its target to its degree of effectiveness. The total number of ASCMs and/or decoys that can be defeated by the SAM defenses of the force is called the SAM Defensive Capacity. It is the product of:

- Average Number of SAM Fire Control Channels
- Average SAM Defensive Readiness
- Number of Ships in the force

Because the salvo exchange is instantaneous, the Anti-Surface Warfare Salvo Model does not allow reassignment of the SAM fire control channels to engage more than one ASCM or decoy. Therefore, the total Number of SAM fire control channels for the defending force represents the upper limit for SAM Defensive Capacity.

SAM Defensive Readiness is the extent to which a defending warship takes defensive actions up to its designed combat potential. It has a value between zero and one. A value of one represents full readiness; a value closer to zero implies low defensive readiness.

SAM Defense Leakers

The Anti-Surface Warfare Salvo Model injects additional realism into the Hughes' model by accounting for ASCMs that leak through the SAM defense. Some number of ASCMs will penetrate this layer because of imperfect SAM engagements or fire control ambiguity. Also, SAM defenses can be saturated with more missiles and decoys than can be defended. In this case, some ASCMs will go uncontested because there are not enough fire control channels to engage all of the missiles and decoys that make up the salvo. Both Tiah and Wissel incorporate the effects of leakers based on the defensive posture of the forces. Each force can find itself in one of two SAM defensive postures:

1. A Strong SAM Defense occurs when a defending force has more SAM fire control channels than there are ASCMs and decoys in the salvo. This occurs when the SAM Defensive Capacity of one force is greater than, or equal to, the Apparent Striking Power of the other. For this posture, a SAM fire control channel is assigned to every ASCM and offensive decoy, and each SAM engagement will be successful to its degree of effectiveness. The Combat Power surviving a Strong SAM Defensive Posture is the product of:

- (1 - SAM Defense Effectiveness)
- Apparent Striking Power of the attacking force
- Ratio of Striking Power to Apparent Striking Power of the attacking force

SAM Defense Effectiveness is the probability that a SAM fire control channel will defeat an ASCM.

The Ratio of Striking Power to Apparent Striking Power accounts for the offensive decoys that are being engaged by SAMs.

2. A Weak SAM Defense occurs when the attacking force saturates the SAM defenses with more ASCMs and offensive decoys than there are SAM fire control channels. This occurs when the Apparent Striking Power of one force is greater than the Defensive Capability of the other. In this situation, a certain number of ASCMs and decoys will not be contested, and the remaining ASCMs and offensive decoys will be results in some leakers due to less than perfect SAM defenses. The number of uncontested ASMC is the difference between:

- Apparent Striking Power of the attacking force
- SAM Defensive capability of the defending force

The number of leakers that penetrate the SAM defense is the product of:

- (1 - SAM Defense Effectiveness)
- SAM Defensive capability of the defending force

The Combat Power surviving a Weak Defense Posture is the sum of uncontested ASCMs and the number of leakers that penetrate SAM defensive capabilities, scaled by the Ratio of Striking Power to Apparent Striking Power

Soft Kill Measures

Defense against ASCMs will most likely include soft-kill assets. Wissel introduce the decoy dilution effect on the defensive power. The calculations for diluting the number of ASCMSs away from the defending force are based upon a simple premise that each decoy will be just as effective at drawing a missile as a targeted ship. This means that if there is one decoy and one ship 50% of the missiles will be drawn to a decoy and 50% will continue to the ship. With two decoys this percentage decreases to 33% for each decoy and ship. In the general case, the reduction of the Combat Power is the ratio of the number of ships in the defending force to the sum of the number of ships and the number of decoys launched.

It is assumed that offensive decoys are not seduced by defensive soft kill measures.

Point Defense

ASCM point defense includes gun systems and short range missile systems. Since these systems are short range, each point defense fire control channel will be assigned to a single ASCM. Another words, the maximum number of ASCMs than can be defeated by point defense systems equals the number of point defense fire control channels of the defending force.

The Anti-Surface Warfare Model assumes that point defenses systems will discriminate ASCMs form offensive decoys. Another words, offensive decoys do not have terminal homing capabilities and do not pose a threat to point defense systems.

The allocation of point defense fire control channels to ASCMs is identical to the SAM defense modeling. A strong point defensive posture is the situation where one force has more point defense fire control channels than there are surviving ASCMs in the salvo. In this case, every ASCM will be assigned to a point defense fire control channel, and each point defense fire control channel will defeat the ASCM to within its degree of effectiveness. Point defense fire control independence is assumed so the number of ASCMs defeated by the point defense systems is the product of:

- Point defense effectiveness
- Number of surviving ASCMS

where point defense effectiveness is the probability that the point defenses of the force will defeat an ASCM when it is engaged.

A weak offensive posture occurs when the surviving number of ASCMs of the attacking force is greater than the number of point defense fire control channels of the defending force. In this case, the number of point defense fire control channels is the limiting factor and some ASCMs will not be engaged by point defenses. For these situations, Defensive Power is defined by the product of:

- Point defense effectiveness
- Point defensive capability

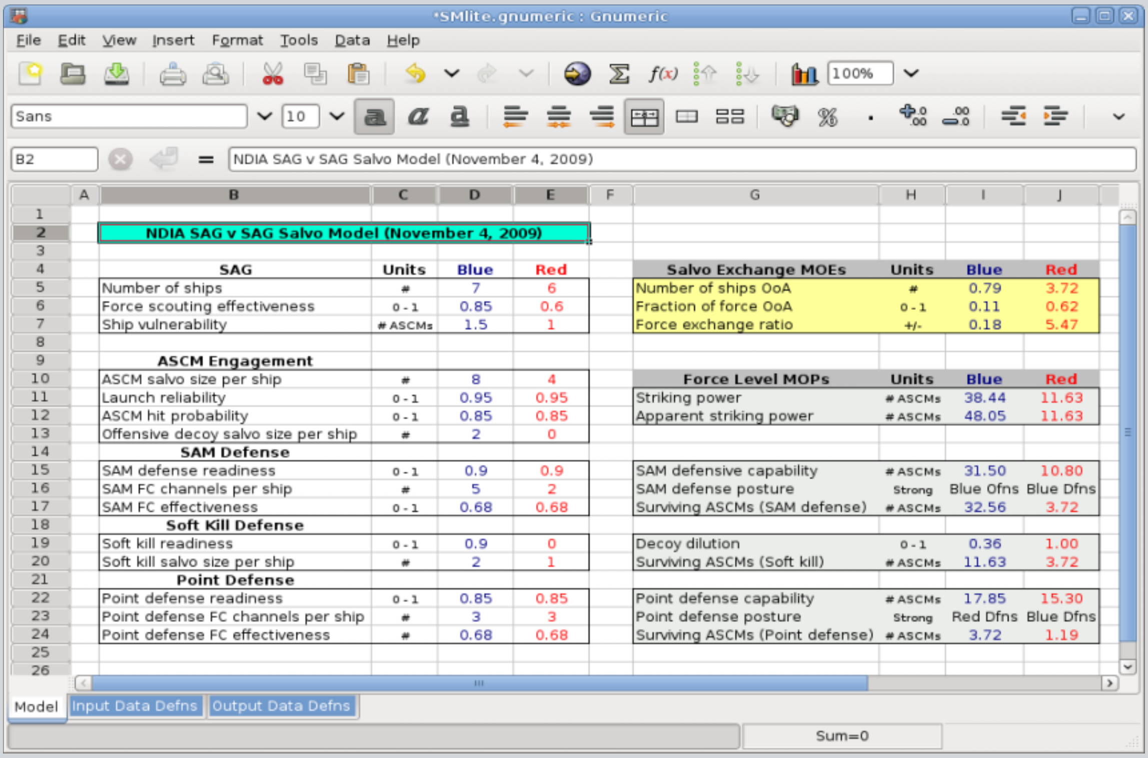
The resulting Combat Power of the forces is the number of ASCMs that survived all of the layer defenses and damage the opposing force.

Anti-Surface Warfare Salvo Model Implementation

The simplicity of Hughes' salvo model, and its many variants, provides fertile grounds for their implementation to facilitate analysis objectives and to model complex battle situations. While slightly more complex than Hughes' model, the basic equations of the Anti-Surface Warfare Salvo Model can be implemented, or coded, as a mathematical function that can be orchestrated to represent highly complex battle situations. Several implementation concepts are worthy of explanation.

The simple application of the Anti-Surface Warfare Salvo Model represents a Red/Blue exchange where both forces attack and defend against the opposing forces. Analysis for this situation is deterministic and straight forward. Initial force data and the sequence of defensive capability directly relates to the number of ship taken out of action for each force. For these analyses, a spreadsheet implementation of the Salvo Model lends itself to data clarity and cause and effect analysis.

The Anti-Surface Warfare Salvo Model, implemented in a spreadsheet format, is shown in Figure 2. The spreadsheet architecture groups all of data for the opposing forces in a vertical format on the left hand side of the sheet. The left hand side of the spread sheet organizes all of the data that is computed by the model, including the measures of effectiveness for the salvo exchange. For the sake clarity, the opposing forces are called Blue and Red. All of the data associated with the Blue and Red forces are colored coded. The spreadsheet equations handle the dueling aspects of the equations and all interactions between the Blue and Red data.



NDIA SAG v SAG Salvo Model (November 4, 2009)

SAG	Units	Blue	Red
Number of ships	#	7	6
Force scouting effectiveness	0 - 1	0.85	0.6
Ship vulnerability	# ASCMs	1.5	1
ASCM Engagement			
ASCM salvo size per ship	#	8	4
Launch reliability	0 - 1	0.95	0.95
ASCM hit probability	0 - 1	0.85	0.85
Offensive decoy salvo size per ship	#	2	0
SAM Defense			
SAM defense readiness	0 - 1	0.9	0.9
SAM FC channels per ship	#	5	2
SAM FC effectiveness	0 - 1	0.68	0.68
Soft Kill Defense			
Soft kill readiness	0 - 1	0.9	0
Soft kill salvo size per ship	#	2	1
Point Defense			
Point defense readiness	0 - 1	0.85	0.85
Point defense FC channels per ship	#	3	3
Point defense FC effectiveness	#	0.68	0.68

Salvo Exchange MOEs	Units	Blue	Red
Number of ships OoA	#	0.79	3.72
Fraction of force OoA	0 - 1	0.11	0.62
Force exchange ratio	+/-	0.18	5.47
Force Level MOPs	**Units**	**Blue**	**Red**
Striking power	# ASCMs	38.44	11.63
Apparent striking power	# ASCMs	48.05	11.63
SAM defensive capability	# ASCMs	31.50	10.80
SAM defense posture	Strong	Blue Ofns	Blue Dfns
Surviving ASCMs (SAM defense)	# ASCMs	32.56	3.72
Decoy dilution	0 - 1	0.36	1.00
Surviving ASCMs (Soft kill)	# ASCMs	11.63	3.72
Point defense capability	# ASCMs	17.85	15.30
Point defense posture	Strong	Red Dfns	Blue Dfns
Surviving ASCMs (Point defense)	# ASCMs	3.72	1.19

Figure 2 Anti-Surface Warfare Salvo Model

The spreadsheet implementation of the Salvo Model is a useful tool to analyze a Red/Blue exchange where both forces attack and defend against the opposing force. The results obtained from this implementation scheme are deterministic and represent a unique situation (battle) frozen in time and force capability.

If on the other hand, analysis requirements call for expected force capabilities, the Salvo Model can be implemented stochastically. This is done by ....

References

Jeffrey R. Cares, (2004) "AN INFORMATION AGE COMBAT MODEL," Alidade Incorporated

Yao Ming Tiah, (2007) "AN ANALYSIS OF SMALL NAVY TACTICS USING A MODIFIED HUGHES' SALVO MODEL, " Naval Post Graduate School Thesis

Hughes, W.P., Jr., (2000). "Fleet Tactics: Theory and Practice," Naval Institute Press, Annapolis, MD.

Nicholas E. Wissel (2008) "SURFACE COMBATANT READINESS TO CONFRONT A SEA CONTROL NAVY," Naval Post Graduate School Thesis

Michael D. Johns, (2000) "HETEROGENEOUS SALVO MODEL FOR THE NAVY AFTER NEXT," Naval Post Graduate School Thesis

McGunnigles, J., (1999) "AN EXPLORATORY ANALYSIS OF THE MILITARY VALUE OF INFORMATION AND FORCE," Naval Post Graduate School Thesis